# ECO206 Notes

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# 1 Budget constraints

# 2 Preference and Utility

## 2.1 Bundle notation

We denote a bundle as

$$(x_1^A, x_2^A, \ldots, x_n^A) \in \mathbb{R}_+^n$$

In short form this is

$$A \equiv (x_1^A, x_2^A)$$

If we strictly prefer bundle $A$ over $B$, we say

$$A \succ B$$

If we weakly prefer bundle $A$ over $B$, we say

$$A \succeq B$$

If $A$ and $B$ are indifferent, we say

$$A \sim B$$

### 2.1.1 Complete preferences

Completeness – Individuals can always make comparison between bundles

$$\forall A, B, (A \succeq B) \text{ or } (B \succeq A) \text{ or } (A \sim B)$$

### 2.1.2 Transitive preferences

Transitivity – preferences are internally consistent

$$(A \succeq B) \text{ and } (B \succeq C) \implies A \succeq C$$

We consider consumers to be rational; that is, consumers have both complete and transitive preferences.

### 2.1.3 Monotonicity

Monotonicity – More is better or at least not worse
If we have

$$x_i^B \geq (>) x_i^A \forall i$$

then

$$B \succeq (\succ) A$$

The key here is for **all** i; every good in one bundle must be $\geq$ the corresponding good in the other bundle for this bundle to be preferred.

### 2.1.4 Convexity

Convexity – The averages are better than the extremes
Suppose we have $A \sim B$. Then for any $a \in [0,1]$, we have

$$(ax_1^A + (1-a)x_1^B, ax_2^A + (1-a)x_2^B) \succeq A$$

$$(ax_1^A + (1-a)x_1^B, ax_2^A + (1-a)x_2^B) \succeq B$$

# 3 Constrained optimization

## 3.1 MRS and opportunity cost

The best bundle is the most preferred bundle (the one that gives the highest utility).

### 3.1.1 MRS vs OC

The $|OC|$ tells us that we are able to trade $y$ for $x$ at a rate of $\frac{p_x}{p_y}$ given the current market prices.
The $|MRS|$ tells us that we are willing to give up $\frac{MU_x}{MU_y}$ units of $y$ for one unit of $y$.

Ideally, given a budget constraint and utility function, we want to buy the best bundle where $|OC| = |MRS|$.

- If $|MRS| < |OC|$ at the current bundle, this means are willing to give up less units of $y$ than we have to at the current market prices
- If $|MRS > |OC|$ at the currentl bundle, this means we are willing to give up more units of $y$ than we have to at the current bundle

Clearly, the most optimal is when $|MRS| = |OC|$ because we won't have to buy more/less units of a good to maximize our utility. The point where $|MRS| = |OC|$ is called a **tangency point** because at this point, the budget constraint is a tangent line to an indifference curve.

## 3.2 Method of Lagrangians

A Lagrange function is a function $\mathcal{L}(x_1, x_2, \lambda)$. If our objective is the maximize **utility** given a **budget constraint**, then we want to maximize $u(x_1, x_2)$ subjected to $p_1x_1 +$

$p_2x_2 \leq I$. That is, we set up

$$\mathcal{L} = u(x_1, x_2) + \lambda(I - p_1x_1 - p_2x_2)$$

### 3.2.1 First order conditions

We set each partial derivative of $\mathcal{L}$ to 0, thus we have

$$\mathcal{L} = u(x_1, x_2) + \lambda(I - p_1x_1 - p_2x_2)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_1} = 0 &\implies \frac{\partial u}{\partial x_1} - \lambda p_1 = 0 \implies \frac{\partial u}{\partial x_1} = \lambda p_1 \\ \frac{\partial \mathcal{L}}{\partial x_2} = 0 &\implies \frac{\partial u}{\partial x_2} - \lambda p_2 = 0 \implies \frac{\partial u}{\partial x_2} = \lambda p_2\end{aligned}$$

We notice that we this implies

$$|MRS| = \frac{\lambda p_1}{\lambda p_2} = \frac{p_1}{p_2}$$

which gives the desired result. The partial derivative $\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_1x_1 - p_2x_2$ tells us what the best bundle from the set of affordable bundles is, so we plug in the above result into $\frac{\partial \mathcal{L}}{\partial \lambda}$.

### 3.2.2 When can/can't we use Lagrangians?

**Definition** (Essential Good)**.** Any bundle with 0 units of the good is as good as having no goods at all.

**Definition** (Corner solution)**.** Optimal choice has 0 units of one of the goods

**Definition** (Interior solution)**.** Strictly positive amount of both goods in the optimal choice

The First Order Conditions are necessary and sufficient to identify a consumer's optimal consumption bundle if

- There are no flat spots on the curve
- All goods are essential (meaning the optimal solution isn't when one of the goods has a quantity of 0)
- Curve is convex

Basically, always watch out for corner solutions and non-convexities if the curve isn't homothetic + convex.

# 4 Demand and substitution

**Definition** (Demand)**.** Optimal bundles as a a function of income and prices

$$x_1 = \varphi(p_1, p_2, I), x_2 = \psi(p_1, p_2, I)$$

- To solve for demand, always compare $MRS$ vs $MRS$ and keep $p_x$ and $p_y$ as variables
- Use a piecewise demand function if the comparison $MRS = OC$ will return a negative quantity for one of the goods
  - If perfect subs, set up $MRS > OC$, $MRS = OC$ and $MRS < OC$
  - If quasilinear, set up $MRS = \varphi(x_1)$ and then set up a Lagrangian and solve for $MRS = \frac{p_1}{p_2} = OC$
  - Won't split into cases if optimal solutions are interior solutions (ie: homothetic and convex)

## 4.1 Demand shifters

- Change in $p_x$ while holding $I$ and $p_y$ fixed will shift the demand curve of $x$ along the curve
- Change in $I$ or $p_y$ while holding $p_x$ fixed will shift the demand curve of $x$ outwards

# 5 Income and substitution effects

## 5.1 Income effects

**Definition** (Income effect)**.** Change in behaviour arising from a change of only income keeping $|OC|$ constant

- Observing parallel shifts of the budget constraint

**Normal goods** Goods that see an increase in demand when the consumer's income increases*

$$\frac{dx}{dI} > 0$$

**Inferior goods** Goods that see a decrease in demand when the consumer's income increases

$$\frac{dx}{dI} < 0$$

In the case of quasilinear preferences, the income effect ***does not exist*** for one of the goods because $MRS$ is a function of one of the goods only

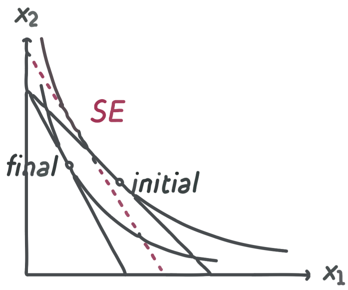
**Example**

Given $u(x_1, x_2) = \ln(x_1) + x_2$, which is clearly quasilinear, we have $MRS = \frac{1}{x_1}$. With a budget constraint $p_1x_1 + p_2x_2 = I$, note that setting $MRS = OC$, which is the demanded bundle, we have that the demand function for $x_1$ is $x_1 = \frac{p_2}{p_1}$, which is independent of $I$. Clearly, changing $I$ will not affect demand for $x_1$ at all.

## 5.2 Substitution effect

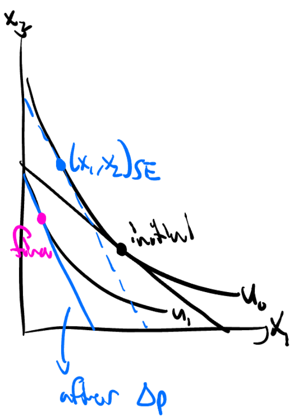
Suppose due to a price increase we want to give cash to people so demand increases. However, due to the income effect, giving more people means their incomes increase, so then when they buy a different bundle (assuming normal goods), then utility does not remain constant. The key question for substitution effect: **how can we give cash to people so they stay on the initial utility curve with new prices?** To solve for

$$min_{x_1, x_2} p_1^{final} x_1 + p_2^{final} x_2 \text{ subject to } u(x_1, x_2) = U^{initial}$$




the substitution effect bundle, keep the utility constant and the change in prices.

- Note that the total effect of increasing prices is the sum of the income and substitution effects



### 5.3 Compensated demand

The main difference between regular and compensated demands are with what we hold constant: for regular demand, we solve

$$\min_{x_1,x_2} u(x_1, x_2) - \lambda(I - p_1x_1 - p_2x_2)$$

For compensated demand, we solve

$$\min_{x_1,x_2} p_1x_1 + p_2x_2 + \lambda(\overline{U} - u(x_1, x_2))$$

When solving for substitution effect, we let $\overline{U}$ be the initial utility of the initial bundle at the initial prices.

- Note that the regular demand and compensated demands are the same curves if preferences are quasilinear

## 6 Labour market

### 6.1 Labour supply

- Same as with a goods market, except now our goods are leisure ($l$) with a price (wage) $w$, and composite goods ($c$) with a price of 1
- We take an endowment $L$ split between leisure and hours worked $h$, so $L = h + l$, or equivalently, $h = L - l$
- Budget constraint is
$$c + wl = wL + M$$
where $M$ is a fixed income (exogenous) and $wL$ is the maximum amount of money that can be made by allocating 0 of $L$ to leisure (endogenous income)
- Optimizing, we solve
$$\max_{l,c} u(l, c) \text{ by } c + wl \leq wL + M \implies MRS = OC \implies \frac{\frac{\partial u}{\partial l}}{\frac{\partial u}{\partial c}} = w$$

### 6.2 Income and substitution effects in the labour market

If wage increases, then

1. SE: Consumer wants to buy less leisure since they need to work more, so $h \uparrow$
2. IE: Since $h \uparrow$, then $I \uparrow$ so

(a) If leisure is normal (ie: $\frac{dl}{dI} > 0$), then $l \uparrow$ and $h \downarrow$

(b) If leisure is inferior (ie: $\frac{dl}{dI} < 0$), then $l \downarrow$ and $h \uparrow$

**Definition** (Giffen Good)**.** An **inferior good** with $|IE| > |SE|$

- So, we have $w \uparrow \implies h \uparrow \implies I \uparrow \implies l \downarrow \implies h \uparrow$ where the second hours worked increase is larger than the initial

### 6.3 Taxes on wages

- Treat the same as a reduction in wage

## 7 Consumer surplus

**Definition** (Consumer surplus)**.** The difference between WTP and OC in a chosen bundle quantified in $$ terms

- We do a comparison between a good $x$ and let the $y$-axis be the composite good
  - Since $y$ is the composite good, then $|MRS| = |MWTP| = p_x = |OC|$

The motivation is: holding utility constant at $\overline{u}$, what happens if we move along the $\overline{u}$ IC. If we don't hold utility constant, then MRS can literally be anything depending on the IC curve, so we'd have a variable consumer surplus. Thus, we consider the compensated demand curve for consumer surplus.

$$CS = \int_0^\infty h(p_x, p_y = 1, \overline{u})dp$$

where $h(p_x, p_y, \overline{u})$ is the **compensated demand curve**.

### 7.1 Compensating/Equivalent variation

Define the expenditure function

$$E(p_1, p_2, \overline{u}) = p_1 h_1(p_1, p_2, \overline{u}) + p_2 h_2(p_1, p_2, \overline{u})$$

Difference between CV and EV:

- CV → New prices, initial utility

$$CV = E(p_1^{\text{final}}, p_2^{\text{final}}, \overline{u}_{\text{initial}}) - I$$

  CV is the amount of compensation a consumer needs at new prices to maintain initial utility

- EV $\to$ Initial prices, new utility

$$EV = E(p_1^{\text{initial}}, p_2^{\text{initial}}, \overline{u}_{\text{final}}) - I$$

EV is the amount of compensation a consumer needs at old prices to choose $\overline{u}_{\text{final}}$

Note that we only focus on compensated demand because the CD curve has no income effect

### 7.2 Deadweight Loss

**Definition** (Deadweight Loss)**.** Loss in surplus that could be reclaimed by someone in the economy due to a substitution effect

We use EV to make the comparison of money that could've been made if money had just been taken from consumers.

$$\begin{aligned} DWL &= |T - L| \\ &= |\text{Tax} - \text{Lump sum tax revenue}| \\ &= |T - EV| \end{aligned}$$

## 8 Risk and uncertainty

Axes are accident state vs safe state

- There is probability $\delta$ of an accident happening

Budget constraints are insurance contracts, which is the set of all affordable insurance contracts
Consumers pay a premium $p$ in both states and get a benefit $b$ in only the $A$ state
Let $E = (e_A, e_S)$ denote the outcome of choice **without** insurance and $(x_A, x_S)$ be an insurance bundle.

$$\begin{aligned} x_A &= e_A - p + b = e_A + p\left(\frac{1-\gamma}{\gamma}\right) \\ x_S &= e_S - p \end{aligned}$$

### 8.1 Expected value of a gamble

$$\begin{aligned} EV &= \delta x_A + (1-\delta) x_S \\ &= \delta e_A + (1-\delta) e_S + \delta b - p \end{aligned}$$

where $b$ and $p$ are 0 when $(x_A, x_S) = (e_A, e_S)$. If EV without insurance is equal to EV with insurance, we say insurance is **fair**.

**Definition** (Expected utility)**.** Made before uncertainty is revealed

$$u(x_A, x_S) = \delta u_A(x_A) + (1-\delta)u_S(x_S)$$

$$MRS = -\frac{\delta}{1-\delta}\frac{\frac{\partial u}{\partial x_A}}{\frac{\partial u}{\partial x_S}}$$

**Definition** (State independent)**.** The value of money does not depend on the state, so $u_A(x) = u_S(x) = u(x)$ and $MRS = -\frac{\delta}{1-\delta}$

**Definition** (State dependent)**.** The value of money depends on the state, so $u_A(x) \neq u_S(x)$ and $MRS = -\frac{\delta}{1-\delta}\frac{\frac{\partial u}{\partial x_A}}{\frac{\partial u}{\partial x_S}}$

**Definition** (Certainty equivalent)**.** The amount of money the consumer gets for sure that gives $u(x) = u(e_A, e_S) \implies x_A = x_S = x_{CE}$ so $\overline{u} = u(x_{CE}, x_{CE}) = u_{e_A, e_S}$. Basically, certainty equivalence is just the insurance contract that gives utility equivalent to as if the consumer does not insure at all ("risk free")

## 8.2 Risk preferences

Where $E(x) = \delta u_A(EV) + (1-\delta)u_S(EV)$ and $EV$ is the expected value of the gamble,
If $u(E(x)) < u(x_A, x_S)$, the consumer is risk loving.
If $u(E(x)) = u(x_A, x_S)$, the consumer is risk neutral.
If $u(E(x)) > u(x_A, x_S)$, the consumer is risk averse.

## 8.3 Insurance choices

To make a choice of insurance, we solve

$$\max_{b,p} u(x_A, x_S) \text{ subject to } p = \gamma b$$
$$\max_{b,p} \delta u(e_A - p + b) + (1-\delta)u(e_S - p)$$

Assuming risk aversion,

$$\delta\frac{\partial u}{\partial x_A}\left(\frac{1-\gamma}{\gamma}\right) - (1-\delta)\frac{\partial u}{\partial x_S} = 0$$
$$\frac{\delta}{1-\delta}\frac{\frac{\partial u}{\partial x_A}}{\frac{\partial u}{\partial x_S}} = \frac{1-\gamma}{\gamma}$$

If risk neutral, treat it as a perfect substitution type of question. If risk loving, then corner solutions are the solutions.
Full insurance is when $x_A^* = x_S^*$ after insurance choices.

# 9 Production and cost minimization

## 9.1 Single variable production functions

For production, we have a choice over $l$ and $k$ (labour and physical capital).

- In the short run, we conduct single variable analysis (fix $l$ or $k$), so the production function is $x = f(l, \bar{k})$ or $x = f(\bar{l}, k)$
- In the long run, conduct multivariable analysis (varying $l$ and $k$), so the production function is $x = f(l, k)$

**Definition** (Marginal product)**.** How much extra units of output can we produce by increasing $k$ or $l$ by 1 unit.

$$MP_l = \frac{\partial f}{\partial l} \qquad MP_k = \frac{\partial f}{\partial k}$$

Diminishing $MP$ means $\frac{\partial^2 f}{\partial l^2} < 0$ or $\frac{\partial^2 f}{\partial k^2} < 0$

## 9.2 Two input production functions

**Definition** (Isoquants)**.** Equivalent to what the IC is to consumer preferences. Similarly, $MRTS$ is equivalent to what $MRS$ is.

### 9.2.1 Returns to scale

- How much output changes if we increase all inputs proportionately

3 types:

1. IRS: $f(\lambda l, \lambda k) > \lambda f(l, k)$
2. CRS: $f(\lambda l, \lambda k) = \lambda f(l, k)$
3. DRS: $f(\lambda l, \lambda k) < \lambda f(l, k)$

## 9.3 2-step profit maximization

### 9.3.1 Step 1: Cost minimization

There are 3 types of costs:

- Fixed → does not vary with output level
- Sunk → unavoidable costs; none in the long run (sunk costs are paid off in the LR)
- Variable → varies with output level

Define the cost function

$$C(x, w, r) = wl + rk + F$$

where $w$ is wage, $r$ is cost of physical capital, and $F$ is fixed cost. Additionally, define marginal cost as

$$MC = \frac{\partial C}{\partial x}$$

**Short run analysis** Suppose WLOG we fix $k = \overline{k}$. The cheapest way to produce $x = f(l)$ is to minimize $l$ for output level $x = \overline{x}$, so we solve

$$\min_l wl \text{ to } f(l) = \overline{x} \implies l = f^{-1}(x) \implies C(\overline{x}, w) = wl(\overline{x}) = wf^{-1}(x)$$

In actuality, we're solving

$$\min_l wl + r\overline{k} \text{ to } \overline{x} = f(l, \overline{k})$$

**Long run analysis** We're solving

$$\min_{l,k} wl + rk \text{ to } \overline{x} = f(l, k)$$

which gives us our **conditional demand functions**:

$$l(\overline{x}, w, r) \qquad k(\overline{x}, w, r)$$

**Definition** (Isocosts)**.** A curve that contains all input bundles that cost the same (same as the budget constraint from consumer preferences), defined by the equation

$$\overline{C} = wl + rk + F$$

We solve this equation similarly to how we solve for optimal solutions in consumer preferences: deal with non-tangencies the same way we do in utility maximization.
After deriving the conditionals, plug back into

$$C(\overline{x}, w, r) = wl(\overline{x}, w, r) + rk\overline{x}, w, r)$$

In the long run, the production function is the lower envelope of the SR curves

- The SR cost curve slops upwards since there's only one input changing: more input $\implies C \uparrow$
- The LR cost curve shape is determined by RTS

### 9.3.2 Step 2: Maximize difference between revenue and costs

Define

$$\pi(x) = xp(x) - C(x)$$

to be the profit function. To solve, we set $MR = p = MC(x)$ on an upward sloping part of the $MC(x)$ function $\rightarrow$ if $p = MC$ on more than one part of the function, we take $x$ to be the largest $x$ that is a point of intersection on the upward sloping of $MC$.

- If $|\varepsilon_D| = \infty$ for the firm's products, then we're in perfect competition, and the converse holds

In perfect competition, we assume $p$ is constant since everyone in the market is a **price taker**; in later sections, when firms gain market power, $p$ is a function of $x$ and not necessarily constant.

- **ALWAYS CHECK REVENUE $\geq$ AVOIDABLE COSTS**
  - In the SR, costs that don't have to be paid if $x = 0$

$$xp \geq VC \implies p \geq AVC = \frac{VC}{x}$$

  - In the LR, this means all costs (since sink costs are treated as paid in the LR)

$$xp \geq TC \implies p \geq ATC = \frac{TC}{x}$$

  - If a firm's profits fall below this threshold, then the firm either
    - Produces $x = 0$ in the SR and makes $\pi = 0$
    - Exits the market in the LR

## 9.4 One-step profit maximization

- We can't use this method is we have CRS: $f(\lambda l, \lambda k) = \lambda f(l, k)$

The good thing about the one-step method is that it directly finds **unconditional input demands**

$$l(p, w, r) \qquad k(p, w, r)$$

that are *not* dependant on $p$ (notice the conditionals are dependent on $x$ instead of $p$). This method also outputs a supply function but **must still check revenue $\geq$ avoidable**. In this method, the problem we solve is

$$\max_{l,k} xp - wl - rk \text{ subject to } x = f(l, k)$$

# 10 Market equilibrium

**Definition** (Market demand)**.** Horizontal sum of all individual demand curves

$$x_m(p) = \sum_i x_i(p)$$

**Definition** (Market supply)**.** In the short run, market supply is the horizontal sum of all firm supplies of firms in the market (since the number of firms is fixed in the short run due to no free entry/exit)
In the long run, there is free entry/exit, so the number of firms is not fixed

- Firms are only in the market if $p \geq \min AVC$
- As $p \uparrow$, more firms have incentive to enter so $x \uparrow$, which drives $p \downarrow$ to $\min AC$ so LR market supply is perfectly elastic at $\min AC$

In the SR, market EQ is at market supply = market demand
In the LR, $p_{mkt} = \min AC$, so we want $q_D = q_S$. Additionally, since firm output is determined by $\min AC$, then the number of firms of the same type is equal to

$$\text{No. firms} = \frac{q_S}{\text{Firm output}}$$

# 11 Welfare and distortions

## 11.1 Taxes

**Definition** (Statutory incidence)**.** The person who is legally required to pay the tax

**Definition** (Economic incidence)**.** Change in effective prices or how tax is split between consumers and produces once we look at market price changes

There are two types of taxes: % of market prices collected or a per unit tax of $\tau$ collected per unit sold. Taxation gives us two effective prices, $p_D$ and $p_S$, but still only 1 market price. The following table shows how to comptue taxes:

| **Statutory incidence** | **Market price** | **Relation between $p_D$ and $p_S$** |
|---|---|---|
| **Consumer** | $p_S$ | $p_D = p + \tau$ or $p_D = (1 + \%)p_S$ |
| **Producer** | $p_D$ | $p_S = p - \tau$ or $p_S = (1 - \%)p_D$ |

Note the relation

$$p_D = p_S + \tau$$

is present in both situations. This shows that when we have a per-unit tax, economic incidence does not depend on statutory.

## 11.2 Price controls and quotas

### 11.2.1 Price ceiling

At a price ceiling, the quantity traded is $\min\{x_S, x_D\}$ and the DWL is determined by $\varepsilon$ and price ceiling level. With a price ceiling $p^*$, $p$ is not allowed to rise above $p^*$, so it's only effective below market EQ (if above, then the market price would just be the equilibrium price).

### 11.2.2 Price floor

At a price floor, the quantity traded is $\min\{x_D, x_S\}$ and the DWL is determined by $\varepsilon$ and price floor level. With a price floor $p^*$, $p$ is not allowed to dip below $p^*$, so it's only effected below market EQ (if below, then the market price would just be the equilibrium price).

The difference between taxes and price controls is with a tax, $x_D(p_D) = x_S(p_S)$. However, with a price control, $x_D(p) \neq x_S(p)$, so the excess demand or supply which leads to additional sources of DWL.

# 12 General equilibrium

## 12.1 Edgeworth boxes

1. Flip one person's axes
2. More graphs until overlap at $E_A = E_B$
    - Dimensions of the box are given by $x_A + x_B$ by $y_A + y_B$

Mutually beneficial trades are allocations that are at least as good as the initial endowment for both individuals. An efficient allocation is a bundle where improving upon the bundle would make one person better off but the other worse off when $IC_A = IC_B$;

if there exists a gap betweeen $IC_A$ and $IC_B$, then there can also be a more efficient allocation. The set of all efficient allocations is called the contract curve.

### 12.1.1 Contract curve

To compute the contract curve, set $\overline{u} = u_A$. Then, plug

$$x_1^A = e_1 - x_1^B$$
$$x_2^A = e_2 - x_2^B$$

into $MRS_A = MRS_B$ and solving for $x_2^A = f(x_1^A)$ for example. Define the core to be the intersection between all mutually beneficial allocations and the contract curve.

### 12.1.2 Competitive equilibrium

- The set of prices and an allocation such that at these prices, everyone choose the allocation bundle

To solve,

1. $\max_{x_1,x_2} u_A(x_1, x_2)$ to $p_1x_1 + p_2x_2 \leq p_1e_1 + p_2e_2 \to x_1^A(p_1, p_2), x_2^A(p_1, p_2)$
2. $\max_{x_1,x_2} u_B(x_1, x_2)$ to $p_1x_1 + p_2x_2 \leq p_1e_1 + p_2e_2 \to x_1^B(p_1, p_2), x_2^B(p_1, p_2)$
3. Solve for $p_1, p_2$ at $D = S$
4. Plug in

$$x_1^A + x_1^B = e_1^A + e_2^B$$
$$x_2^A + x_2^B = e_2^A + e_2^B$$

Walras' Law states that if at $p_1^*$, we have $D = S$ for $x_1$, then $D = S$ for $x_2$ as well.

# 13 Monopoly

**A monopolist must sell all products at the same price**

To solve the monopolist supply curve, we solve

$$\max_x xp(x) - C(x)$$

where $p(x)$ is a function of $x$. Since the monopolist has market power, they can set price, so we solve

$$p'(x)x + p(x) = C'(x)$$

where $MR = p'(x)x+p(x)$ is called the marginal revenue. In perfect competition, $MR = p$ since $p$ is constant since everyone is a price taker.

### 13.1 Monopsony

When there's a single buyer of input who has market power. Think of the labour market, suppose there's 1 firm in the labour market looking to hire and extra worker. In order to hire an extra worker, they have to raise wages to be able to afford hiring another worker, however, they must pay all existing workers a higher wage too since they have to pay all workers the same wage. So, we observe $ME = w + w'l$ where $ME$ is marginal expenditure.

## 14 Price discrimination

There are 4 things required to be able to price discriminate:

1. Prevent arbitrary - resale of goods
2. Market power
3. Can segment customers
4. Information about segment MWTPs

### 14.1 3rd degree

- Have info on group + individual demands
- Every group member must pay the same price

We solve

$$\max_{x_A, x_B} p_A(x_A)x_A + p_B(x_B)x_B - C(x_A + x_B)$$

We want $MR_A = MR_B = MC$. With 3rd degree, we can sell to a lower MWTP without lowering price for higher MWTP people, so there's less DWL than with monopoly.

### 14.2 2 part tariff

- Have info on individual demand
- Everyone pays the same fixed and per unit price

We set the fixed fee to be the lower $MWTP$'s CS, or else the lower MWTP people won't buy the product. This captures some CS for the producer. Also have to check what would happen if we cater to only high MWTP, and as price increase, the fixed fee decreases since the more income from higher MWTP people, there's less surplus from the lower.

### 14.3 1st degree

- Have info on individual demand curves
- Everyone gets their own prices $\implies CS = 0$

In first degree, $x_M = x_{EQ}$, so maximal $CS$ is extracted by the monopolist.

### 14.4 2nd degree

- Requires self-selection by the consumer
- 2 options:
  1. Couple lower price with an obstacle that higher valuation consumers find too costly (for example: free version of an app but missing many valuable features compared to the paid version)
  2. Offer different packages and let consumers decide (for example: types of yoga classes at a gym of varying intensity)

For 2nd degree, we solve

$$\max_{x_1,x_2} p_1(x_1)x_1 + p_2(x_2)(x_2 - x_1) - C(x_2)$$

since we have to make $x_2$ total units. The key is, we want to extract maximal consumer surplus from the higher MWTP group by offering them the maximum quantity for a price that offers more benefit than if they pretend to be the lower type. To solve, suppose we offer $q$ to the lower type. Then, compute the benefit from as if the higher type pretends to buy the contract with $q$ quantity, then offer higher type the maximum quantity for a price that's equal to the maximum CS of the higher type $-$ the benefit the net benefit they get by pretending.

## 15 Game theory

**Definition** (Strictly dominant strategy). Payoff from strategy $i >$ payoff from $j$ regardless of other's action

**Definition** (Nash equilibrium). Mutual best response between $A$ and $B$

A Prisoner's Dilemma is when the NE is not the most efficient allocation.

# 16 Oligopoly

## 16.1 Cournot competition

Cournot competition is where firms make simultaneous decisions and pick quantity.
Given market demand

$$p = A - \alpha x_M$$

and market quantity $x_M = x_1 + x_2$ where firm 1 produces $x_1$ and firm 2 produces $x_2$. Given $x_2$, Firm 1 treats it as a constant and as if it's already in the market.

$$p = A - \alpha x_1 - \alpha x_2$$
$$x_1 = \frac{A - \alpha x_2 - p}{\alpha}$$

### 16.1.1 Best response function

A best response function maximizes a firms choices based off what the other firms will do. After solving for the BR functions, we solve for market equilibrium by solving a simultaneous game theory problem.

## 16.2 Stackelberg competition

Stackelberg competition is where firms pick quantity but engage in sequential timing. We solve this problem like we'd solve a sequential game theory problem. Given two firms, suppose Firm 1 enters first and Firm 2 enters second. We first solve for Firm 2's choice. Since when they enter the market, they will know Firm 1's decision, we solve Firm 2's problem similar to how we solved for Cournot competition. For Firm 1, however, solve using backward induction, so we *don't treat $x_2$ as a constant since know how Firm 2 will react to $x_1$*.

## 16.3 Bertrand competition

Firms choose **price**.
Firms in Bertrand competition can either do simultaneous or sequential timing.

### 16.3.1 Simultaneous

Set prices and meet any demand they get; there is no capacity constraint unlike in quantity competition. For Bertrand, there are large swings in the residual demands since consumers will buy at the lowest price. For this reason, there's always an incentive to undercut the other firm if they price greater than $MC$.

**Example**

Suppose $p = 100 - x$ and $MC = 40$, so the monopoly price is 70. For Firm 1, we treat $p_2$ as given. If $p_2 > 70$, then $p_1 = 70$ since all customers will buy $x_1$ so $\pi_1$ will be maximized. If $p_2 < MC$, then $p_2$ will be making negative profits. This means that $p_1$ should price at $MC$ and make 0 profits. Since all customers will buy $x_2$, then since $p_1 = MC$ anyways, Firm 1 will not lose money. If $p_2 \in [40, 70]$, then we price $p_1 = p_2 - \varepsilon$; they price just below Firm 2 and capture the entire market. So, the best response function of Firm 1 is given by

$$p_1 = \begin{cases} 70 & p_2 > 70 \\ p_2 - \varepsilon & p_2 \in [40, 70] \\ 40 & p_2 < 40 \end{cases}$$

### 16.4 Sequential

In sequential timing, there is continuous undercutting until $\pi_1 = \pi_2 = 0$ at $p = MC$; once a firm prices below MC, the other will price at MC. To solve this problem, we solve similar to how we solve Stackelberg.

- Second mover: Solve for BR from simultaneous
- First mover: Set $p = MC$ in anticipation and then solve for EQ at $p = MC$

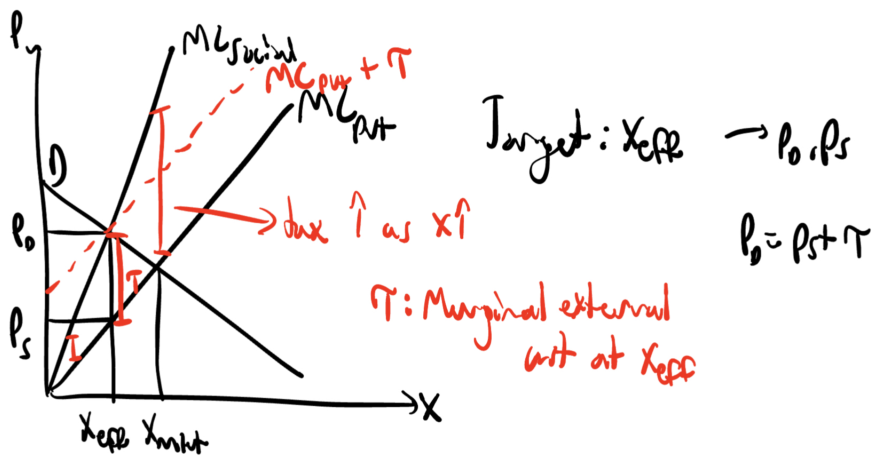
## 17 Externality

**Definition** (Externality)**.** Costs or benefits imposed on a third party external to the activity

With externality, market efficiency is when social benefit = social cost. Observing externalities in an edgeworth box, we need a starting endowment. Then, the Coase Theorem states that as long as transaction costs are low, it doesn't matter how property rights are assigned as outcome of externality is independent of who gets the rights.

### 17.1 Pigouvian taxes and subsidies

- Act as a way to get market to social efficient outcome




**Example**

Market demand is $x = 120 - p$, market supply is $x = p$, there's an externality on production of $0.5x^2$. What is the Pigouvian tax?
Since no externality on market demand, $MPB = MSB = 120 - x$. Since $MSC = MPC + MEC = x + x = 2x$, we solve for $x_{eff}$:

$$120 - x = 2x \implies x = 40$$

Solving for $p_D$, we have $p_D = 120 - 40 = 80$ and solving for $p_S$, we have $p_S = 40$, so since

$$p_D = p_S + \tau$$

we have $\tau = 40$.

## 17.2 Cap and trade

- Consider pollution vouchers

Cap: government determines the optimal quantity of pollution, so they distribute vouchers that give the right to pollute
Trade: the firms buy/sell vouchers, which creates a market for vouchers and results in voucher price $r$

### 17.2.1 Voucher market

In the voucher market, supply is perfectly inelastic (set amount by the government). In the output market, $p = \beta x = MC$ without pollution. If a firm needs to buy vouchers to pollute $\delta$ units, then $MC \uparrow$ and shifts market supply up, so market supply becomes

$$p = MC = \beta x + r\delta$$

This acts in the same way a Pigouvian tax would on an externality on production.